

Dual Wilson Loop and Infrared Monopole Condensation in Lattice QCD in the Maximally Abelian Gauge

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Abstract

Using the SU(2) lattice QCD, we formulate the dual Wilson loop and study the dual Higgs mechanism induced by monopole condensation in the maximally abelian (MA) gauge, where QCD is reduced into an abelian gauge theory including the electric current j_μ and the monopole current k_μ . After the abelian projection in the MA gauge, the system can be separated into the photon part and the monopole part corresponding to the separation of j_μ and k_μ , respectively. We study here the monopole part (the monopole-current system), which is responsible to the electric confinement. Owing to the absence of electric currents, the monopole part is naturally described using the dual gluon field B_μ without the Dirac-string singularity. Defining the dual Wilson loop from the dual gluon B_μ , we find the perimeter law of the dual Wilson loop in the lattice QCD simulation. In the monopole part in the MA gauge, the inter-monopole potential is found to be flat, and can be fitted as the Yukawa potential in the infrared region after the subtraction of the artificial finite-size effect on the dual Wilson loop. From more detailed analysis of the inter-monopole potential considering the monopole size, we estimate the effective dual-gluon mass $m_B \simeq 0.5\text{GeV}$ and the effective monopole size $R \simeq 0.2\text{fm}$. The effective mass of the dual gluon field at the long distance can

be regarded as an evidence of “infrared monopole condensation”.

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I. INTRODUCTION

Quantum chromo-dynamics (QCD) is the fundamental theory of the strong interaction [1–7] and is an $SU(N_C)$ nonabelian gauge theory described by the quark field q and the gluon field A_μ as

$$\mathcal{L}_{QCD} = -\frac{1}{2}\text{tr}(G_{\mu\nu}G^{\mu\nu}) + \bar{q}(i\not{D} - m_q)q, \quad (1)$$

where $G_{\mu\nu}$ is the $SU(N_C)$ field strength $G_{\mu\nu} \equiv \frac{1}{ie}[D_\mu, D_\nu]$ with the covariant-derivative operator $D_\mu = \partial_\mu + ieA_\mu$. Due to the asymptotic freedom, which is one of the most important features in QCD the gauge-coupling constant of QCD becomes small in the ultraviolet region [1–3]. Accordingly, the perturbative QCD can describe the high-energy phenomena like the Bjorken scaling and the hadron jet properties [4–6].

On the other hand, in the low-energy region, the QCD-coupling constant becomes large, and there arise the nonperturbative-QCD (NP-QCD) phenomena such as color confinement and dynamical chiral-symmetry breaking corresponding to the strong-coupling nature. These NP-QCD phenomena are extremely difficult to understand in the analytical manner from QCD, and have been studied by using the effective models [6] or the lattice QCD simulation [7]. Here, the lattice QCD Monte Carlo simulation is the numerical calculation of the QCD partition functional, and it is one of the most reliable methods directly based on QCD. In fact, the lattice QCD simulations well reproduce nonperturbative quantities such as the quark static potential, the chiral condensate $\langle \bar{q}q \rangle$ and low-lying hadron masses [7].

Recently, the lattice QCD simulation has shed light on the confinement mechanism in terms of the dual-superconductor picture, which was proposed by Nambu, 't Hooft and Mandelstam in the middle of 1970's [8–10]. In this scenario, quark confinement can be understood with the dual version of the superconductivity. In the ordinary superconductor, the Meissner effect occurs by condensation of the Cooper-pair with the electric charge. Consider the existence of the magnetic charges with the opposite sign immersed in the superconductor, then the magnetic flux is squeezed like a tube between the magnetic charges, and the

magnetic potential between them becomes linear as the result of the Meissner effect [11]. In the dual-superconductor scenario, the QCD vacuum is assumed as the dual version of the superconductor, and the dual Meissner effect brings the one-dimensional flux squeezing [12] between the quark and the anti-quark, which leads to the linear confinement potential [4–8,13].

The dual Higgs mechanism, however, requires “color-magnetic monopole condensation” as the dual version of electric condensation in the superconductor, although QCD does not include the color-magnetic monopole as the elementary degrees of freedom. On the appearance of magnetic monopoles from QCD, ’t Hooft showed that QCD is reduced to an abelian gauge theory with magnetic monopoles by taking the abelian gauge, which fixes the partial gauge symmetry $SU(N_C)/U(1)^{N_C-1}$ through the diagonalization of a gauge-dependent variable [14]. Here, the monopole appears as the topological object corresponding to the nontrivial homotopy group $\pi_2(SU(N_C)/U(1)^{N_C-1})=Z_\infty^{N_C-1}$.

As for the irrelevance of off-diagonal gluons, recent lattice QCD studies show the abelian dominance [15] for the NP-QCD phenomena in the maximally abelian (MA) gauge [16,17]. For instance, confinement [18,19] and dynamical chiral-symmetry breaking [20,21] are almost described only by the diagonal gluon component, in the MA gauge. Then, taking the MA gauge and removing off-diagonal gluons, the abelian-projected QCD (AP-QCD) is obtained as the abelian gauge theory keeping the NP-QCD features. AP-QCD includes not only the electric current j_μ but also the magnetic current k_μ , and can be decomposed into the monopole part and the photon part corresponding to the separation of k_μ and j_μ , respectively [22,23]. The lattice QCD studies show that only the monopole part is responsible to NP-QCD phenomena [20,23–26] especially to the electric confinement [23–25] in the MA gauge. This is called as the monopole dominance.

In this paper, we concentrate the monopole part (the monopole-current system) in the MA gauge in QCD, and study the dual Higgs mechanism in the QCD vacuum based on the dual gauge formalism [27,28]. To this end, we perform the $SU(2)$ lattice QCD simulation in the MA gauge, and extract the monopole current k_μ as the relevant degrees of freedom

for electric confinement. Then, we calculate inter-monopole potential in the monopole part, (the monopole current system) in QCD to examine monopole condensation, and evaluate the effective mass of the dual gluon field B_μ [27,28].

II. SEPARATION OF AP-QCD INTO THE MONOPOLE PART AND THE PHOTON PART IN THE MA GAUGE

A. MA Gauge Fixing and AP-QCD

Recent studies with the lattice QCD Monte Carlo simulation have revealed the abelian dominance and the monopole dominance in the maximally abelian (MA) gauge for the non-perturbative QCD (NP-QCD) phenomena such as confinement, dynamical chiral-symmetry breaking and instantons [18-26].

In the continuum Euclidean QCD with $N_c = 2$, the MA gauge fixing is defined by the minimizing the total amount of off-diagonal gluons,

$$R_{\text{off}} = \int d^4x [\{A_1^\mu(x)\}^2 + \{A_2^\mu(x)\}^2] = 2 \int d^4x [A_+^\mu(x)A_-^\mu(x)] \quad (2)$$

with

$$A_\mu(x) = \sum_{a=1}^3 A_\mu^a(x) \frac{\tau^a}{2}, \quad (3)$$

$$A_\mu^\pm(x) \equiv \frac{1}{\sqrt{2}} \{A_\mu^1(x) \pm iA_\mu^2(x)\}, \quad (4)$$

by the SU(2)-gauge transformation. In the MA gauge, off-diagonal gluon components, A_μ^1 and A_μ^2 , become as small as possible, and the gluon field A_μ mostly approaches to the abelian gluon field, $A_\mu^{\text{Abel}} \equiv A_\mu^3 \frac{\tau^3}{2}$. In the MA gauge, the SU(2) local symmetry is reduced into the $U(1)_3$ local symmetry with the global Weyl symmetry. Under the residual $U(1)_3$ -gauge transformation with $\omega = \exp(-i\phi \frac{\tau^3}{2}) \in U(1)_3$, the gluon components are transformed as

$$A_\mu^3(x) \rightarrow A_\mu^3(x) + \partial_\mu \phi(x), \quad (5)$$

$$A_\mu^\pm(x) \rightarrow e^{i\phi(x)} A_\mu^\pm(x). \quad (6)$$

Then, in the MA gauge, the diagonal gluon A_μ^3 behaves as the abelian gauge field, while off-diagonal gluons A_μ^\pm behave as charged matter fields in terms of the residual gauge symmetry.

As a remarkable feature of the MA gauge, the abelian dominance holds for the NP-QCD phenomena such as quark confinement and chiral-symmetry breaking [18-21]. Here, we call abelian dominance for an operator \hat{O} , when the expectation value $\langle O[A_\mu] \rangle$ is almost equal to the expectation value $\langle O[A_\mu^{\text{Abel}}] \rangle_{\text{MA}}$, where off-diagonal gluons are dropped off in the MA gauge. For instance, the abelian string tension $\sigma_{\text{Abel}} \equiv \langle \sigma(A_\mu^{\text{Abel}}) \rangle_{\text{MA}}$ in the MA gauge is almost equal to $\sigma_{\text{SU}(2)} \equiv \langle \sigma(A_\mu^{\text{SU}(2)}) \rangle$ as $\sigma_{\text{Abel}} \simeq 0.92\sigma_{\text{SU}(2)}$ for $\beta \simeq 2.5$ in the lattice QCD [24,25]. Thus, NP-QCD phenomena are almost reproduced only by the abelian gluon A_μ^{Abel} , and off-diagonal gluon components A_μ^\pm do not contribute to NP-QCD in the MA gauge. Hence, as long as the infrared physics is concerned, QCD in the MA gauge can be approximated by the abelian projected QCD (AP-QCD), where the SU(2) gluon field A_μ is replaced by the abelian gluon field A_μ^{Abel} . In other word, AP-QCD is the abelian gauge theory keeping essence of NP-QCD, and is extracted from QCD in the MA gauge. Hereafter, we pay attention to the AP-QCD described by A_μ^{Abel} in the MA gauge.

The abelian-projected QCD (AP-QCD) includes not only j_μ but also k_μ . In this subsection, we investigate the general argument on the extended electro-magnetic system including both the electric current j_μ and the magnetic current k_μ . In the extended Maxwell equations with j_μ and k_μ , the field strength $F_{\mu\nu}^{\text{Abel}}$ satisfies as

$$\partial_\mu F_{\mu\nu}^{\text{Abel}} = j_\nu \quad (7)$$

$$\partial_\mu {}^*F_{\mu\nu}^{\text{Abel}} = -\partial_\mu {}^*\xi_{\mu\nu} = k_\nu \quad (8)$$

with ${}^*F_{\mu\nu}^{\text{Abel}} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}^{\text{Abel}}$. In the presence of both j_μ and k_μ , the field strength $F_{\mu\nu}^{\text{Abel}}$ cannot be described by the simple two-form $\partial_\mu A_\nu^{\text{Abel}} - \partial_\nu A_\mu^{\text{Abel}}$ with the regular one-form A_μ^{Abel} [29]. In general, the field strength $F_{\mu\nu}^{\text{Abel}}$ consists of two parts,

$$F_{\mu\nu}^{\text{Abel}} = (\partial \wedge A^{\text{Abel}})_{\mu\nu} + {}^*\xi_{\mu\nu}, \quad (9)$$

where the former part denotes the ordinary two-form and the latter part ${}^*\xi_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\xi_{\alpha\beta}$ denotes the Dirac-string singularity [29]. Here, $\xi_{\mu\nu}$ can be written as

$$\begin{aligned}
\xi_{\mu\nu}(x) &= \frac{1}{n \cdot \partial} (n \wedge k)_{\mu\nu} \\
&= \int d^4y \langle x | \frac{1}{n \cdot \partial} | y \rangle (n_\mu k_\nu(y) - n_\nu k_\mu(y)) \\
&= \int d^4y \theta(x_n - y_n) \delta^3(\mathbf{x}^\perp - \mathbf{y}^\perp) (n_\mu k_\nu(y) - n_\nu k_\mu(y)),
\end{aligned} \tag{10}$$

where $x_n \equiv x_\mu n_\mu$ and $x_\mu^\perp \equiv x_\mu - (x \cdot n)n_\mu$. Here, n_μ is arbitrary four-dimensional unit vector corresponding to the direction of the Dirac string. Thus, in the ordinary description [29] the system includes the singularity as the Dirac string $\xi_{\mu\nu}$, which makes the analysis complicated.

B. Separation of AP-QCD into Photon Part and Monopole Part

To clarify the roles of j_μ and k_μ to the nonperturbative quantities of QCD, we consider the decomposition of AP-QCD into the photon part and the monopole part, corresponding to the separation of j_μ and k_μ . We call this separation into the photon and monopole parts as the “photon projection” and the “monopole projection”, respectively. [26,30].

The field strength $F_{\mu\nu}^{\text{Abel}}$ in AP-QCD is separated into $F_{\mu\nu}^{\text{Ph}}$ in the photon part and $F_{\mu\nu}^{\text{Mo}}$ in the monopole part

$$F_{\mu\nu}^{\text{Abel}} = F_{\mu\nu}^{\text{Ph}} + F_{\mu\nu}^{\text{Mo}}. \tag{11}$$

As the physical requirement, $F_{\mu\nu}^{\text{Ph}}$ and $F_{\mu\nu}^{\text{Mo}}$ satisfies the Maxwell equations as

$$\partial_\mu F_{\mu\nu}^{\text{Ph}} = j_\nu, \quad \partial_\mu {}^*F_{\mu\nu}^{\text{Ph}} = 0, \tag{12}$$

$$\partial_\mu F_{\mu\nu}^{\text{Mo}} = 0, \quad \partial_\mu {}^*F_{\mu\nu}^{\text{Mo}} = k_\nu, \tag{13}$$

respectively. From the requirement (12), there exists the regular vector A_μ^{Ph} is defined so as to satisfying

$$F_{\mu\nu}^{\text{Ph}} = \partial_\mu A_\nu^{\text{Ph}} - \partial_\nu A_\mu^{\text{Ph}} \tag{14}$$

in the photon part. This relation is the same as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ in the ordinary QED.

As for the monopole part, $F_{\mu\nu}^{\text{Mo}}$ cannot be expressed by the simple two-form, but is expressed as

$$\partial_\mu A_\nu^{\text{Mo}} - \partial_\nu A_\mu^{\text{Mo}} = F_{\mu\nu}^{\text{Mo}} + {}^*\xi_{\mu\nu}, \quad (15)$$

where the second term provides the breaking of the Bianchi identity. From Eqs.(9), (11), (14) and (15), one finds the relation satisfy

$$\partial \wedge A^{\text{Abel}} = \partial \wedge (A^{\text{Ph}} + A^{\text{Mo}}), \quad (16)$$

and then the difference between A_μ^{Abel} and $A_\mu^{\text{Ph}} + A_\mu^{\text{Mo}}$ is at most the total differential term as $\partial_\mu \chi$. Hence, we can set

$$A_\nu^{\text{Abel}} = A_\nu^{\text{Ph}} + A_\nu^{\text{Mo}} \quad (17)$$

by taking a suitable gauge without loss of generality.

In the practical calculation, the photon part variable A_μ^{Ph} and the monopole part variable A_μ^{Mo} can be obtained as

$$A_\nu^{\text{Ph}} = \square^{-1} j_\nu = \square^{-1} \partial_\mu F_{\mu\nu}^{\text{Abel}}, \quad (18)$$

$$A_\nu^{\text{Mo}} = \square^{-1} \partial_\mu {}^*\xi_{\mu\nu}, \quad (19)$$

where the inverse d'Alembertian \square^{-1} is the nonlocal operator [31],

$$\langle x | \square^{-1} | y \rangle = -\frac{1}{4\pi^2} \frac{1}{(x-y)^2}, \quad (20)$$

which satisfies $\square_x \langle x | \square^{-1} | y \rangle = \langle x | y \rangle = \delta^4(x-y)$.

Let us check that A_μ^{Ph} and A_μ^{Mo} defined in Eqs.(18) and (19), satisfies the physical requirement in Eqs.(14) and (15) First, we consider the photon part with A_μ^{Ph} . Starting from Eq.(18), one easily finds $\partial_\mu A_\mu^{\text{Ph}} = 0$, and hence the first Maxwell equation in Eq.(12) in the photon part can be derived as

$$\begin{aligned} \partial_\mu F_{\mu\nu}^{\text{Ph}} &= \partial_\mu (\partial_\mu A_\nu^{\text{Ph}} - \partial_\nu A_\mu^{\text{Ph}}) = \square A_\nu^{\text{Ph}} - \partial_\nu (\partial_\mu A_\mu^{\text{Ph}}) = \square A_\nu^{\text{Ph}} \\ &= \square \square^{-1} j_\nu = j_\nu. \end{aligned} \quad (21)$$

The second Maxwell equation in Eq.(12) is automatically derived as the Bianchi identity for the two form of A_μ^{Ph} . Thus, the photon part with A_μ^{Ph} defined by Eq.(18) does not include the magnetic current but only includes the electric current.

Second, we consider the monopole part with A_μ^{Mo} . From Eqs.(9), (18) and (19), the sum of A_μ^{Ph} and A_μ^{Mo} can be written as

$$\begin{aligned} A_\mu^{\text{Ph}} + A_\mu^{\text{Mo}} &= \square^{-1} \partial_\alpha (F_{\alpha\mu}^{\text{Abel}} + {}^* \xi_{\alpha\mu}) = \square^{-1} \partial_\alpha (\partial_\alpha A_\mu^{\text{Abel}} - \partial_\mu A_\alpha^{\text{Abel}}) \\ &= A_\mu^{\text{Abel}} - \partial_\mu (\partial_\alpha A_\alpha^{\text{Abel}}), \end{aligned} \quad (22)$$

and hence the two-form of A_μ^{Mo} satisfies the physical requirement (15) as

$$\begin{aligned} (\partial_\mu A_\nu^{\text{Mo}} - \partial_\nu A_\mu^{\text{Mo}}) &= \partial_\mu A_\nu^{\text{Abel}} - \partial_\nu A_\mu^{\text{Abel}} - (\partial_\mu A_\nu^{\text{Ph}} - \partial_\nu A_\mu^{\text{Ph}}) \\ &= F_{\mu\nu}^{\text{Abel}} + {}^* \xi_{\mu\nu} - F_{\mu\nu}^{\text{Ph}} \\ &= F_{\mu\nu}^{\text{Mo}} + {}^* \xi_{\mu\nu}. \end{aligned} \quad (23)$$

Therefore, in the monopole part, the Maxwell equations is derived as

$$\partial_\mu F_{\mu\nu}^{\text{Mo}} = \partial_\mu F_{\mu\nu}^{\text{Abel}} - \partial_\mu F_{\mu\nu}^{\text{Ph}} = 0, \quad (24)$$

$$\partial_\mu {}^* F_{\mu\nu}^{\text{Mo}} = \partial_\mu {}^* F_{\mu\nu}^{\text{Abel}} - \partial_\mu {}^* F_{\mu\nu}^{\text{Ph}} = k_\nu, \quad (25)$$

starting from Eq.(19).

Thus, A_μ^{Ph} and A_μ^{Mo} defined as Eqs.(18), and (19) satisfy the physical requirement Eqs.(14) and (15). Hence, the monopole part carries the same amount of the magnetic current as that is the original abelian sector, whereas it dose not carries the electric current. The situation is just the opposite in the photon part. In the actual lattice QCD simulation, the monopole current k_μ and the electric current j_μ are slightly modified through the monopole and the photon projections, respectively, due to the numerical error on the lattice. However, these differences are negligibly small in the actual lattice QCD simulation. In fact, $k_\mu^{\text{Mo}} \simeq k_\mu$ and $j_\mu^{\text{Mo}} \simeq 0$ hold in the monopole part, and $j_\mu^{\text{Ph}} \simeq j_\mu$ and $k_\mu^{\text{Ph}} \simeq 0$ hold in the photon part within 1% error. Here, we have kept the labels as “*Mo*” and “*Ph*” for the

electric current and the monopole current, and we have used $(k_\mu^{\text{Mo}}, j_\mu^{\text{Mo}})$ and $(k_\mu^{\text{Ph}}, j_\mu^{\text{Ph}})$ for these currents in the monopole part and the photon part, respectively.

As a remarkable fact, lattice QCD simulations show that nonperturbative quantities such as the string tension, the chiral condensate and instantons are almost reproduced only by the monopole part in the MA gauge, which is called as monopole dominance [23-26]. On the other hand, the photon part dose not contribute these nonperturbative quantities in QCD.

Since we are interested in the NP-QCD phenomena, it is convenient and transparent to extract the relevant degrees of freedom for NP-QCD by removing irrelevant degrees of freedom like the off-diagonal gluons A_μ^\pm and the electric current j_μ . Therefore, we concentrate ourselves to the monopole part, which keeps the essence of NP-QCD as confinement.

III. DUAL GAUGE FORMALISM

-DUAL GLUON FIELD AND DUAL WILSON LOOP-

In this section, we study the monopole part of the QCD vacuum using the dual gauge formalism [27,28]. In the MA gauge, the monopole part carries essence of the nonperturbative QCD as the electric confinement. According to the absence of the electric current ($j_\mu = 0$), the Maxwell equation in the monopole part becomes

$$\partial_\mu F_{\mu\nu}^{\text{Mo}} = 0 \quad (26)$$

$$\partial_\mu {}^*F_{\mu\nu}^{\text{Mo}} = k_\nu, \quad (27)$$

where $F_{\mu\nu}^{\text{Mo}}$ denotes the field strength in the monopole part. This system resembles the dual version of QED with $j_\mu \neq 0$ and $k_\mu = 0$, and hence it is useful to introduce the dual gluon field B_μ in the monopole part for the study of the dual Higgs mechanism in QCD [27,28].

The dual gluon field B_μ is defined so as to satisfy the relation

$$\partial_\mu B_\nu - \partial_\nu B_\mu = {}^*F_{\mu\nu}^{\text{Mo}}, \quad (28)$$

which is the dual version of the ordinary relation $F_{\mu\nu} \equiv \partial_\mu A_\nu^{\text{Mo}} - \partial_\nu A_\mu^{\text{Mo}}$ in QED. The interchange between A_μ and B_μ corresponds to the electro-magnetic duality transformation,

$F_{\mu\nu} \leftrightarrow {}^*F_{\mu\nu}$ or $\mathbf{H} \leftrightarrow \mathbf{E}$. Owing to the absence of j_μ , the dual gauge field B_μ can be introduced without the Dirac-string singularity. In the other words, the absence of j_ν is automatically derived as the dual Bianchi identity,

$$j_\nu = \partial_\mu F_{\mu\nu} = \partial_\mu {}^*(\partial \wedge B)_{\mu\nu} = 0. \quad (29)$$

Let us consider the derivation of the dual gauge field B_μ from the monopole current k_μ . Taking the dual Landau gauge $\partial_\mu B_\mu = 0$, the Maxwell equation $\partial_\mu {}^*F_{\mu\nu}^{\text{Mo}} = \partial^2 B_\nu - \partial_\nu(\partial_\mu B_\mu) = k_\nu$ is simply reduced as $\square B_\mu = k_\mu$. Therefore, the dual gluon field B_μ is obtained by using the inverse d'Alembertian \square^{-1} as

$$B_\nu(x) = \square^{-1} k_\nu \quad (30)$$

or equivalently

$$B_\nu(x) = \int d^4y \langle x | \square^{-1} | y \rangle k_\nu(y) = -\frac{1}{4\pi^2} \int d^4y \frac{1}{(x-y)^2} k_\nu(y). \quad (31)$$

Thus, the monopole part is described by the monopole current k_μ and the dual gluon B_μ in the regular manner based on the dual gauge formalism.

In the dual superconductor picture in QCD, k_μ and B_μ correspond to the Cooper-pair and the photon in the superconductor, respectively. The Cooper-pair and the photon are essential degrees of freedom which bring the superconductivity. In the superconductor, the photon field A_μ gets the effective mass as the result of Cooper-pair condensation, and this leads to the Meissner effect. Accordingly, the potential between the static electric charges becomes the Yukawa potential $V_Y(r) \propto \frac{e^{-mr}}{r}$ in the ideal superconductor obeying the London equation. Similarly, the dual gluon B_μ is expected to be massive in the the monopole-condensed system, and the mass acquirement of B_μ leads to the dual Meissner effect. In other words, the acquirement of dual gluon mass m_B reflects monopole condensation, and brings electric confinement. Hence, we can investigate the dual Higgs mechanism in QCD by evaluating the dual gluon mass m_B , which is estimated from the inter-monopole potential.

To estimate the interaction between the monopoles, we propose the dual Wilson loop W_D [27,28]. The dual Wilson loop W_D is defined by the line-integral of the dual gluon field $B_\mu \equiv B_\mu^3 \frac{\tau^3}{2}$ along a closed loop C ,

$$W_D(C) \equiv \frac{1}{2} \text{tr} \exp (ie_M \oint_C B_\mu dx_\mu) = \text{Re}[\exp (i\frac{e_M}{2} \oint_C B_\mu^3 dx_\mu)], \quad (32)$$

which is the *dual version of the abelian Wilson loop*

$$W_{\text{Abel}}(C) \equiv \frac{1}{2} \text{tr} \exp (ie \oint_C A_\mu^{\text{Abel}} dx_\mu) = \text{Re}[\exp (i\frac{e}{2} \oint_C A_\mu^3 dx_\mu)]. \quad (33)$$

Using the Stokes theorem, the dual Wilson loop $W_D(C)$ is rewritten as

$$W_D(C) = \frac{1}{2} \text{tr} \exp (ie_M \int_S {}^*F_{\mu\nu}^{\text{Mo}} dS_{\mu\nu}), \quad (34)$$

with the dual gauge field strength $F_{\mu\nu}^{\text{Mo}}$. The dual Wilson loop $W_D(R \times T)$ describes the interaction between the monopole-pair with the test magnetic charges $\frac{e_M}{2}$ and $-\frac{e_M}{2}$. Here, these test magnetic charges are pair-created at $t = 0$ and are pair-annihilated at $t = T$ keeping the spatial distance R for $0 \leq t \leq T$. As T goes to infinity, the dual Wilson loop $\langle W_D(R \times T) \rangle$ means the interaction between the static monopole and anti-monopole with the separation of the distance R . The inter-monopole potential is obtained from the dual Wilson loop as

$$V_M(R) = - \lim_{T \rightarrow \infty} \frac{1}{T} \ln \langle W_D(R \times T) \rangle \quad (35)$$

in a similar manner to the extraction of the inter-quark potential from the Wilson loop [4-7]. To summarize here, for the investigation of the dual Higgs mechanism in QCD, we have introduced the dual gluon field B_μ and the dual Wilson loop in the monopole part of the AP-QCD in the MA gauge. In the next section, we consider the practical procedure on the calculation of the dual Wilson loop and the inter-monopole potential in the lattice QCD formalism.

IV. DUAL GAUGE FORMALISM ON THE LATTICE

We study the dual Wilson loop and the inter-monopole potential in the MA gauge using the SU(2) lattice QCD Monte Carlo simulation. The lattice QCD simulation is the direct calculation of the QCD partition functional using the Monte Carlo method. The physical expectation value of an observable $O[A_\mu]$ is numerically obtained by averaging its value over all gauge configurations with the weight factor $\exp(-S[A_\mu])$, where $S[A_\mu]$ denotes the lattice QCD action in the Euclidean metric [7].

In the lattice gauge formalism with the lattice spacing a , the SU(2) link variable is defined by $U_\mu(s) \equiv \exp(iaeA_\mu(s)) = \exp(iaeA_\mu^a(s)\tau^a/2) \in \text{SU}(2)$, where e and $\tau^a/2$ denote the QCD gauge coupling and the generator of the SU(2) group, respectively. The standard lattice QCD action in the gauge sector is defined by

$$S^{\text{L}} = \beta \sum_{s,\mu,\nu} \left[1 - \frac{1}{2N_c} \text{tr} \{ U_{\mu\nu}(s) + U_{\mu\nu}^\dagger(s) \} \right], \quad \beta \equiv \frac{2N_c}{e^2} \quad (36)$$

using the plaquette variable $U_{\mu\nu}(s) \equiv U_\mu(s)U_\nu(s+\mu)U_\nu^\dagger(s+\nu)U_\mu^\dagger(s)$. In the continuum limit $a \rightarrow 0$, the plaquette $U_{\mu\nu}(s)$ becomes $\exp[ia^2 G_{\mu\nu}(s)]$, and hence S^{L} coincides with the continuum QCD action

$$S = \int d^4x \frac{1}{2} \text{tr}(G_{\mu\nu}G_{\mu\nu}). \quad (37)$$

Thus, the QCD system is described by the link variable $U_\mu(s) \in \text{SU}(N_c)$ instead of the gauge field $A_\mu(x) \in \text{su}(N_c)$ in the lattice formalism.

Here, we consider the extraction of the abelian-projected QCD (AP-QCD) from the lattice QCD. In the SU(2) lattice formalism, the MA gauge fixing is achieved by maximizing

$$R = \frac{1}{2} \text{tr} \sum_{s,\mu} [\tau^3 U_\mu(s) \tau^3 U_\mu^\dagger(s)] = \sum_{s,\mu} \left[1 - 2 \left(\{U_\mu^1(s)\}^2 + \{U_\mu^2(s)\}^2 \right) \right] \quad (38)$$

by the SU(2) gauge transformation,

$$U_\mu \rightarrow U_\mu^{\text{MA}} = V(s)U_\mu(s)V^\dagger(s+\hat{\mu}), \quad (39)$$

where $V(s)$ and $V(s + \hat{\mu})$ are the gauge functions located at the starting and end points of the link variable $U_\mu(s)$. In this gauge, the absolute value of off-diagonal components $U_\mu^1(s)$ and $U_\mu^1(s)$ are forced to be small as possible using the gauge degrees of freedom.

In accordance with the Cartan decomposition, the $SU(2)$ link variable $U_\mu(s)$ is factorized as

$$U_\mu^{\text{MA}}(s) = M_\mu(s)u_\mu(s) , \quad (40)$$

$$M_\mu \equiv \exp[i(\theta_\mu^1 \tau^1 + \theta_\mu^2 \tau^2)], \quad u_\mu(s) \equiv \exp[i(\theta_\mu^3 \tau^3)], \quad (41)$$

where $u_\mu \in U(1)_3$ and $M_\mu \in SU(2)/U(1)_3$ correspond to the diagonal part and the off-diagonal part of the gluon field, respectively. In the continuum limit, the angle variable θ_μ^a goes to the gluon field A_μ^a as $\theta_\mu^a \rightarrow \frac{1}{2}eaA_\mu^a$. The off-diagonal factor $M_\mu(s)$ is rewritten as

$$M_\mu(s) = e^{i(\theta_\mu^1 \tau^1 + \theta_\mu^2 \tau^2)} = \begin{pmatrix} \cos \theta_\mu & -\sin \theta_\mu e^{-i\chi_\mu} \\ \sin \theta_\mu e^{i\chi_\mu} & \cos \theta_\mu \end{pmatrix} \quad (42)$$

with

$$\theta_\mu \equiv \text{mod}_{\pi/2} \sqrt{(\theta_\mu^1)^2 + (\theta_\mu^2)^2} \in [0, \frac{\pi}{2}], \quad \chi \equiv \tan^{-1} \frac{\theta_\mu^1}{\theta_\mu^2}. \quad (43)$$

Under the abelian gauge transformation with $v(s) \in U(1)_3$, $M_\mu(s)$ and $u_\mu(s)$ are transformed as

$$M_\mu(s) \rightarrow M_\mu^v(s) = v(s)M_\mu(s)v^\dagger(s), \quad (44)$$

$$u_\mu(s) \rightarrow u_\mu^v(s) = v(s)u_\mu(s)v^\dagger(s + \hat{\mu}), \quad (45)$$

to keep the form of Eq.(41) for $M_\mu^v \in SU(2)/U(1)_3$ and $u_\mu^v \in U(1)_3$. Then, $M_\mu(s)$ behaves as the charged matter field and the abelian link-variable

$$u_\mu(s) = \begin{pmatrix} e^{i\theta_\mu^3} & 0 \\ 0 & e^{-i\theta_\mu^3} \end{pmatrix} \quad (46)$$

behaves as a abelian gauge field with respect to the residual abelian gauge symmetry.

In the lattice QCD, the abelian dominance is expressed as $\langle O(U_\mu) \rangle \simeq \langle O(u_\mu) \rangle_{\text{MA}}$ for the infrared quantities such as σ and $\langle \bar{q}q \rangle$ in the MA gauge. The abelian projection is performed by the replacement of $U_\mu(s) \rightarrow u_\mu(s)$, and then the abelian-projected QCD (AP-QCD) on the lattice is described only with the abelian link-variable $u_\mu(s)$, which is the diagonal part of the SU(2) link-variable $U_\mu^{\text{MA}}(s)$ in the MA gauge.

Next, we consider the separation of AP-QCD into the monopole part and the photon part in the lattice formalism. Using the diagonal gluon component $\theta_\mu^3(s)$, the abelian field strength $\theta_{\mu\nu}^{\text{Abel}}$ is defined by

$$\theta_{\mu\nu}^{\text{Abel}} = \text{mod}_{2\pi}(\partial_\mu \theta_\nu^3 - \partial_\nu \theta_\mu^3) \quad (47)$$

$$= \partial_\mu \theta_\nu^3(s) - \partial_\nu \theta_\mu^3(s) + 2\pi n_{\mu\nu}(s), \quad (48)$$

where the former part denotes the ordinary two-form and $n_{\mu\nu}(s) \in \mathbf{Z}$ corresponds to the Dirac string on the lattice [22]. In the lattice formalism, the photon part $\theta_\mu^{Ph}(s)$ and the monopole part $\theta_\mu^{Mo}(s)$ are obtained from $\theta_{\mu\nu}^{\text{Abel}}(s)$ and $2\pi n_{\mu\nu}(s)$, respectively

$$\theta_\mu^{Ph}(s) = -\{\square^{-1} \partial_\nu \theta_{\mu\nu}^{\text{Abel}}\}(s) \quad (49)$$

$$\theta_\mu^{Mo}(s) = -2\pi\{\square^{-1} \partial_\nu n_{\mu\nu}\}(s), \quad (50)$$

using the inverse d'Alembertian \square^{-1} on the lattice [22-28]: The diagonal gluon component $\theta_\mu^3(s)$ is found to be decomposed as

$$\theta_\mu^3(s) = \theta_\mu^{Ph}(s) + \theta_\mu^{Mo}(s) \quad (51)$$

in the Landau gauge, $\partial_\mu \theta_\mu^3(s) = 0$.

The field strengths, $\theta_{\mu\nu}^{Ph}$ in the photon part and $\theta_{\mu\nu}^{Mo}$ in the monopole part are given as

$$\theta_{\mu\nu}^{Ph} = \text{mod}_{2\pi}(\partial_\mu \theta_\nu^{Ph} - \partial_\nu \theta_\mu^{Ph}) \quad (52)$$

$$\theta_{\mu\nu}^{Mo} = \text{mod}_{2\pi}(\partial_\mu \theta_\nu^{Mo} - \partial_\nu \theta_\mu^{Mo}) \quad (53)$$

on the lattice. In the continuum limit $a \rightarrow 0$, these field strengths becomes as

$$\theta_{\mu\nu}^{Ph} \rightarrow \frac{1}{2}a^2 e F^{Ph} \quad (54)$$

$$\theta_{\mu\nu}^{Mo} \rightarrow \frac{1}{2}a^2 e F^{Mo}. \quad (55)$$

Using the dual field strength $*\theta_{\mu\nu}^{Mo} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\theta_{\rho\sigma}^{Mo}$, the dual Wilson loop is expressed as

$$W_D(C) = \frac{1}{2}\text{tr} \exp \left(i \int_S *\theta_{\mu\nu}^{Mo} dS_{\mu\nu} \right) \quad (56)$$

using the Stokes theorem on the lattice.

To summarize, we show the procedure on the derivation of the dual Wilson loop from lattice QCD as follows:

1. We generate the SU(2) gauge configurations $\{U_\mu(s)\}_i$ using the Monte Carlo method for the lattice QCD.
2. We carry out the gauge transformation, $U_\mu(s) \rightarrow U_\mu^{\text{MA}}(s)$, so as to satisfy the MA gauge fixing condition with the minimization of R .
3. The SU(2) link-variable $U_\mu(s)$ is factorized as $U_\mu(s) = M_\mu(s)u_\mu(s)$, and the abelian projection is performed by the replacement of $U_\mu(s) \in \text{SU}(2)$ by the abelian link-variable $u_\mu(s) = \exp\{i\theta_\mu^3(s)\tau^3\} \in \text{U}(1)_3$.
4. The two-form of the diagonal gluon component is decomposed as $\partial_\mu\theta_\nu^3(s) - \partial_\nu\theta_\mu^3(s) = \theta_{\mu\nu}^{\text{Abel}}(s) + 2\pi n_{\mu\nu}(s)$ with the abelian field strength $\theta_{\mu\nu}^{\text{Abel}} \in (-\pi, \pi]$ and the Dirac string $2\pi n_{\mu\nu} \in 2\pi\mathbf{Z}$.
5. The abelian gauge field θ_μ^3 is decomposed as $\theta_\mu^3 = \theta_\mu^{Mo} + \theta_\mu^{Ph}$ with the photon part θ_μ^{Ph} and the monopole part θ_μ^{Mo} , which are obtained from $\theta_{\mu\nu}^{\text{Abel}}$ and $n_{\mu\nu}$ by way of Eqs.(49) and (50) using the inverse d'Alembert operator \square^{-1} in the Landau gauge.
6. Using the field strength $\theta_{\mu\nu}^{Mo}$ in the monopole part, the dual Wilson loop $\langle W_D(C) \rangle_{\text{MA}}$ and the inter-monopole potential $V_M(r)$ are calculated with Eqs.(56) and (35). Then, the effective dual-gluon mass m_B is estimated from the inter-monopole potential $V_M(r)$.

Thus, we investigate the dual Higgs mechanism in QCD by extracting the dual Wilson loop $\langle W_D(C) \rangle_{\text{MA}}$ and the inter-monopole potential $V_M(r)$ from the gauge configurations obtained in the lattice QCD simulations.

V. LATTICE QCD RESULTS FOR INTER-MONOPOLE POTENTIAL AND DUAL GLUON MASS

In this section, we show the numerical result of the lattice QCD simulation. For the study of the dual Higgs mechanism in QCD, we calculate the dual Wilson loop $W_D(R, T)$ and the inter-monopole potential $V_M(r)$ in the monopole part (the monopole-current system) in QCD in the MA gauge using the SU(2) lattice with 20^4 and $\beta = 2.2 \sim 2.3$. All measurements are performed at every 100 sweeps after a thermalization of 5000 sweeps using the heat-bath algorithm. The physical unit or the lattice spacing a is determined so as to reproduce the string tension $\sigma = 1 \text{ GeV/fm}$ for each β , *e.g.* $a = 0.199 \text{ fm}$ for $\beta = 2.3$. We prepare 100 samples of gauge configurations. These simulations have been performed using the super-computer SX-4 at Osaka University.

The dual Higgs mechanism is characterized by the effective-mass acquirement of the dual gluon B_μ , which is brought by monopole condensation. To examine the effective mass of B_μ , we calculate the inter-monopole potential $V_M(r)$ from the dual Wilson loop $\langle W_D(R, T) \rangle_{\text{MA}}$ obtained in the lattice QCD. As shown in Fig.1, the dual Wilson loop $\langle W_D(R, T) \rangle_{\text{MA}}$ seems to obey the perimeter law rather than the area law for large loops with $I, J \geq 3$. Since the dual Wilson loop $\langle W_D(R, T) \rangle_{\text{MA}}$ satisfies the perimeter law as

$$\ln \langle W_D(R \times T) \rangle_{\text{MA}} \simeq -2(R + T) \cdot \alpha \quad (57)$$

for large R and T , the inter-monopole potential becomes constant 2α in the infinite limit of T ,

$$V_M(R) \rightarrow \lim_{T \rightarrow \infty} \frac{2\alpha}{T} R + 2\alpha = 2\alpha. \quad (58)$$

In the actual lattice QCD calculation, however, we have to take a finite length of T , and hence the linear part $(2\alpha/T)R$ slightly remains as a lattice artifact [28]. Therefore, we have to subtract this lattice artifact $(2\alpha/T)R$ for evaluating of the inter-monopole potential $V_M(R)$ from $\langle W_D(R, T) \rangle$ in the lattice QCD simulation. Here, α can be estimated from the slope of the dual Wilson loop $\ln \langle W_D(R \times T) \rangle_{\text{MA}}$ for large R and T for each lattices. We obtain $\alpha = 0.141 \pm 0.025 \text{ GeV}$ from the dual Wilson loop in Fig.1(b) with $\beta = 2.3$.

After the subtraction of the lattice artifact $(2\alpha/T)R$, we consider the slope of the inter-monopole potential $V_M(r)$ in the lattice QCD. As shown in Fig.2, the inter-monopole potential $V_M(r)$ is short-ranged and flat in comparison with the linear inter-quark potential with string tension $\sigma = 1 \text{ GeV/fm}$. Now, we try to apply the Yukawa potential $V_Y(r)$,

$$V_Y(r) = -\frac{(e/2)^2}{4\pi} \frac{e^{-m_B r}}{r} \quad (59)$$

to the inter-monopole potential $V_M(r)$. As shown in Fig.3, the inter-monopole potential can be fitted by the Yukawa potential $V_Y(r)$ in the long distance region, and we evaluate the dual gluon mass as $m_B \simeq 0.5 \text{ GeV}$.

Finally, we consider the possibility of the monopole size effect, because the QCD monopole is expected to be a soliton like object composed of gluons. In fact, from the recent lattice QCD study, the QCD monopole includes large off-diagonal gluon components near its center even in the MA gauge [28,30,32], and the off-diagonal gluon richness would provide the “effective size” of the QCD monopole similar to the ’t Hooft-Polyakov monopole [1-4]. We introduce the effective size R_M of the QCD-monopole, and assume the Gaussian-type distribution of the magnetic charge around its center,

$$\rho(\mathbf{x}; R_M) = \frac{1}{(\sqrt{\pi} R_M)^3} \exp\left(\frac{-|\mathbf{x}|^2}{R_M^2}\right). \quad (60)$$

Since the monopole part is an abelian system, simple superposition on B_μ is applicable like the Maxwell equation. Therefore, the inter-monopole potential with the effective monopole size R_M is expected to be

$$V(\mathbf{x}; R_M) = -\frac{(e/2)^2}{4\pi} \int d^3x_1 \int d^3x_2 \rho(\mathbf{x}_1; R_M) \rho(\mathbf{x}_2; R_M) \frac{\exp(-m_B |\mathbf{x} - \mathbf{x}_1 + \mathbf{x}_2|)}{|\mathbf{x} - \mathbf{x}_1 + \mathbf{x}_2|}, \quad (61)$$

or equivalently

$$V(r; R_M) = -\frac{(e/2)^2}{\pi^2 R_M^6} \int_0^\infty dr_1 \int_0^\infty dr_2 e^{-(r_1^2 + r_2^2)/R_M^2} \int_0^\pi d\theta_1 \int_0^\pi d\theta_2 \sin \theta_1 \sin \theta_2 \times \frac{\exp[-m_B \sqrt{\{\sqrt{(r - r_2 \cos \theta_2)^2 + (r_2 \sin \theta_2)^2} - r_1 \cos \theta_1\}^2 + (r_1 \sin \theta_1)^2}]}{\sqrt{\{\sqrt{(r - r_2 \cos \theta_2)^2 + (r_2 \sin \theta_2)^2} - r_1 \cos \theta_1\}^2 + (r_1 \sin \theta_1)^2}}, \quad (62)$$

where $r \equiv |\mathbf{x} - \mathbf{y}|$ is the distance between the two monopole centers. We apply the Yukawa-type potential $V(r; R_M)$ to the inter-monopole potential $V_M(r)$ in Fig.3. The potential $V(r; R_M)$ with the effective monopole size $R_M = 0.21\text{fm}$ seems to fit the lattice data of $V_M(r)$ in the whole region of the distance r .

Thus, we estimate the dual gluon mass $m_B \simeq 0.5\text{GeV}$ and the effective monopole size $R_M \simeq 0.2\text{fm}$ by evaluating the inter-monopole potential $V_M(r)$ from the dual Wilson loop $W_D(R, T)$ in the monopole part in the MA gauge. In the long-distance region, we find the effective-mass acquirement of the dual gluon B_μ , which is essential for the dual Higgs mechanism in the dual superconductor scenario. This result suggests “infrared monopole condensation” or monopole condensation in the long-scale description of the QCD vacuum. The monopole size R_M would provide a critical scale [30-32] for the nonperturbative QCD in terms of the dual Higgs theory, because the QCD-monopole structure such as off-diagonal gluons [28,30] should be considered at the shorter scale than R_M , similar to the structure of the 't Hooft-Polyakov monopole.

VI. SUMMARY AND CONCLUDING REMARKS

To examine the dual superconductor picture for the quark confinement mechanism in the QCD vacuum, we have studied the dual Higgs mechanism in terms of the effective-mass acquirement of the dual gluon field B_μ using the lattice QCD Monte Carlo simulation. In the MA gauge, QCD is reduced to the abelian gauge theory with the color-electric current j_μ and the color-magnetic monopole current k_μ . The abelian-projected QCD, the diagonal part of QCD, can be separated into the photon part and the monopole part corresponding to the

separation of j_μ and k_μ , respectively. Reflecting the abelian dominance and the monopole dominance, the monopole part carries essence of NP-QCD and then is of interest in the MA gauge, so that we have concentrated ourselves to the monopole part (the monopole-current system) in QCD.

In order to investigate the dual Higgs mechanism in QCD, we have introduced the dual gluon field B_μ and have studied its features in the monopole part in the MA gauge in the lattice QCD. Owing to the absence of the electric current, the monopole part resembles the dual version of QED, and hence this part is naturally described by the dual gluon field B_μ without meeting the difficulty on the Dirac-string singularity. In the dual gauge formalism, the dual Higgs mechanism is characterized by the acquirement of the effective mass m_B of the dual gluon field B_μ . Then, to evaluate the dual gluon mass, we have calculated the dual Wilson loop $\langle W_D(R \times T) \rangle_{\text{MA}}$, and have studied the inter-monopole potential $V_M(r)$ in the monopole part in the MA gauge based on the dual gauge formalism by using the lattice QCD simulation.

In the lattice QCD, we have found that the dual Wilson loop obeys the perimeter law for large loops. Considering the finite-size effect of the dual Wilson loop, we have investigated the inter-monopole potential $V_M(r)$, and have found that $V_M(r)$ is short-ranged and flat in comparison with the linear inter-quark potential. Then, we have compared the inter-monopole potential $V_M(r)$ with the Yukawa potential and have estimated the dual gluon mass as $m_B \simeq 0.5\text{GeV}$, which is consistent with the phenomenological parameter fitting in the dual Ginzburg-Landau theory [13]. The generation of the dual gluon mass m_B in the infrared region suggests the realization of the dual Higgs mechanism and monopole condensation in the long-scale description of the QCD vacuum. In this way, we have shown the evidence of “infrared monopole condensation” in the lattice QCD in the MA gauge.

To explain the short-range deviation between the inter-monopole potential $V_M(r)$ and the Yukawa potential, we have considered the effective size R_M of the monopole, since the monopole would be a soliton-like object composed of gluons [28,30-32]. The lattice data of the inter-monopole potential can be well fitted with the Yukawa-type potential $V(r; R_M)$

with the effective size $R_M \simeq 0.2$ fm of the monopole. This monopole size $R_M \simeq 0.2$ fm may provide the critical scale for the dual Higgs theory in QCD, because the monopole structure relating to off-diagonal gluons become visible [28,30-32] and the QCD system cannot be described only with the abelian local field theory at the shorter scale than R_M .

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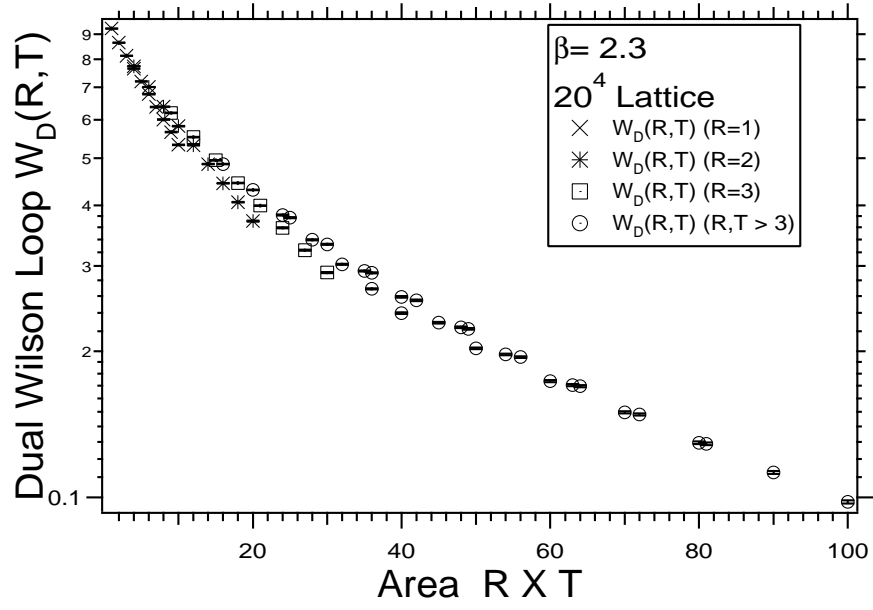
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FIGURES

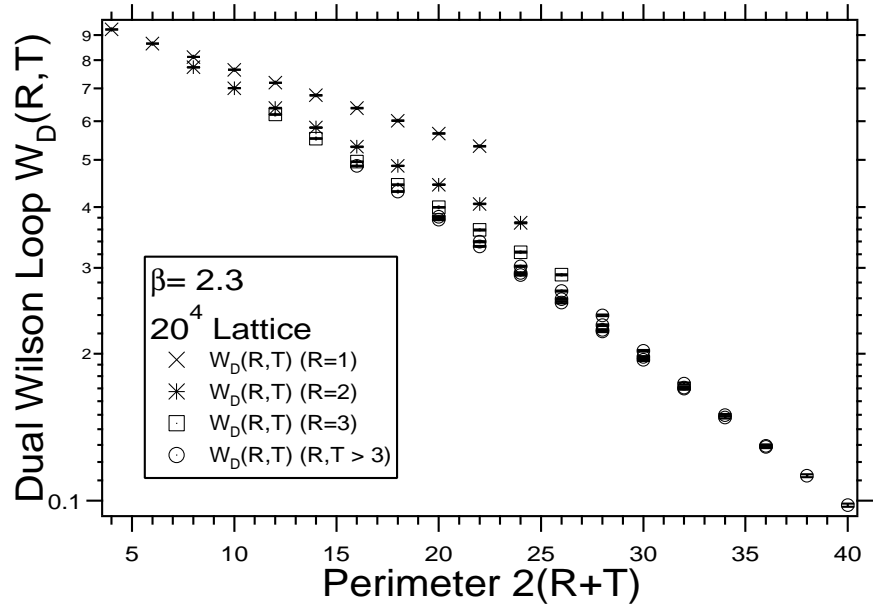
FIG. 1. (a) The dual Wilson loop $\langle W_D(R, T) \rangle_{\text{MA}}$ as the function of its area $R \times T$ (b) $\langle W_D(R, T) \rangle_{\text{MA}}$ as the function of its perimeter $L \equiv 2(R + T)$ in the monopole part in the MA gauge in the SU(2) lattice QCD with 20^4 and $\beta = 2.3$. For large loops as $R, T \geq 3$, $\langle W_D(R, T) \rangle_{\text{MA}}$ seems to obey the perimeter law rather than the area law.

FIG. 2. (a) The inter-monopole potential $V_M(r)$ as the function of the inter-monopole distance r in the monopole part in the MA gauge in the SU(2) lattice QCD with 20^4 lattice and $\beta = 2.2 \sim 2.3$. For comparison, we plot also the linear part of the inter-quark potential $V_q^{\text{linear}}(r) = \sigma r$ with $\sigma = 1.0 \text{ GeV/fm}$ by the straight line. (b) The detail of the lattice QCD data for the inter-monopole potential $V_M(r)$.

FIG. 3. The analysis for the shape of the inter-monopole potential $V_M(r)$ in the SU(2) lattice QCD with 20^4 and $\beta = 2.2 \sim 2.3$. The solid curve denotes the simple Yukawa potential $V_Y(r)$ with the dual gluon mass $m_B = 0.5 \text{ GeV}$. The dotted curve denotes the Yukawa-type potential $V(r; R_M)$ including the magnetic-size effect. The lattice data of the inter-monopole potential $V_M(r)$ seem to be fitted by $V(r; R_M)$ with the effective monopole size $R_M \simeq 0.2 \text{ fm}$ in the whole region of r .

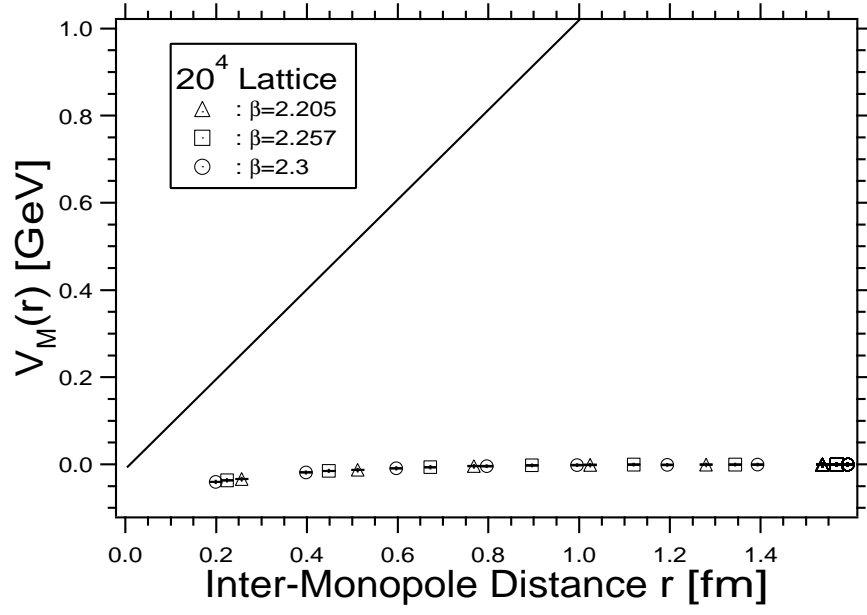


(a)

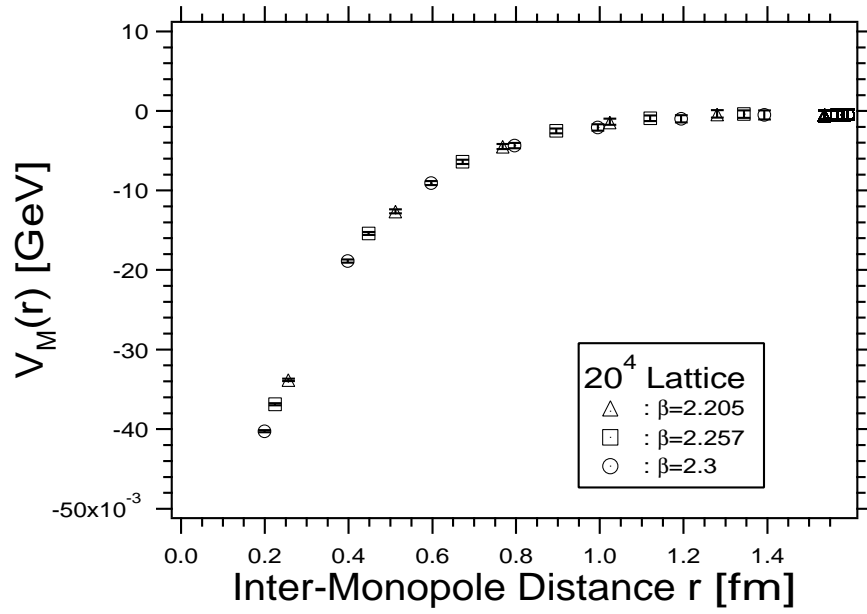


(b)

Figure 1



(a)



(b)

Figure 2

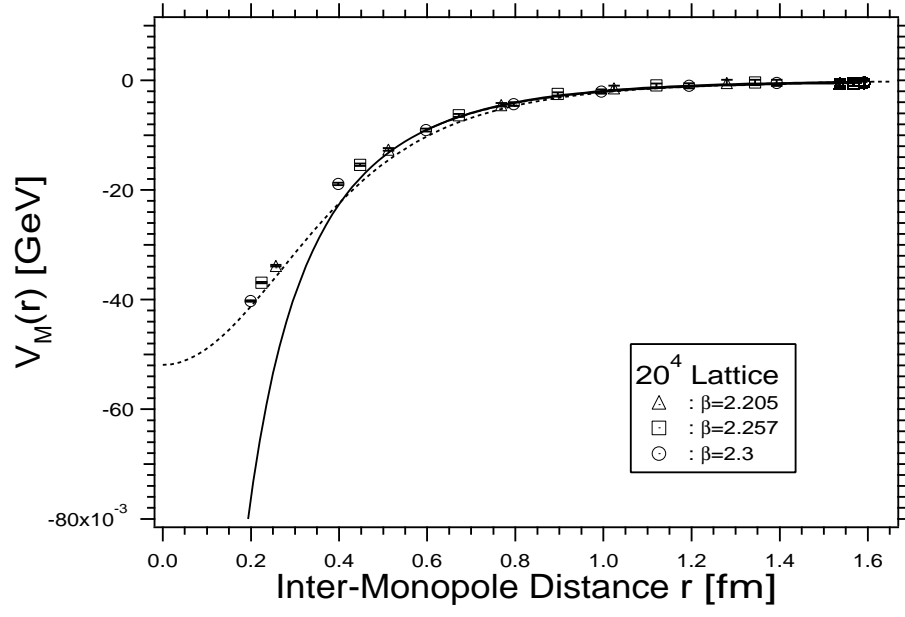


Figure 3